A Quantum Algorithm for Job Shop Problems with Group Theory

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New methods for solving combinatorical optimization problems could be one of the most interesting advances that we receive through quantum computers. In this talk we will cover one of the most prominent examples of such combinatorical optimization problems, named "job shop problem". This problem type deals with the task to organize a number of machines in a manufacture for a certain number of jobs, such that the assignments are in some sense optimal.
The problem statement is equal to the *traveling salesman problem* if one uses only one machine. The traveling salesman problem equals the situation of finding the shortest route between a number of cities.
Although there are various methods for solving this type of problem on classical computers, all of them have exponentially run time in the number of cities. Nowadays it's unclear more then ever before, if quantum computers will ever be suitable to break that barrier substantially. Nevertheless, algorithms like "Groovers search algorithm", which gives a proven square root speed up against all classical solvers of a search problem in an unsorted list, give hopes that there is a quantum algorithm for combinatorical optimization problem, which generates at least a small (provable) speedup.
In this talk, we will discuss a method for describing the set of all solutions of a "job shop problem" with group theory. The main advantage will be, that we find a representation of all solutions $\mathcal{S}$ with one group G and could therefore translate the problem of solving "job shop problems" from

$$\arg\min_{S\in\mathcal{S}} f(S) \quad \text{to} \quad \arg\min_{g\in G} f({}^{g}S),$$

whereby ${}^{g}S$ means a *group action* and f is the function to be optimized. This description has the nice advantage, that solving job shop, therefore means minimizing a functional over a group G. From that point one could find a lot of interesting further directions for solving such problems.
However, the simplest method of encoding by representing a bit with a qubit leads with that problem analysis to a working quantum algorithm (for at least small instances), introduced by Hadfield as the "Alternating Operator Ansatz". This heuristic algorithm leads the talk to the class of variational quantum algorithm.