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Hannover

# Oberseminar für Arithmetische Geometrie und Zahlentheorie

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## Triples, quadruples and quintuples which are $D(n)$ -sets for several $n$ 's.

For an integer  $n$ , a set of distinct nonzero integers  $a_1, a_2, \dots, a_m$  such that  $a_i a_j + n$  is a perfect square for all  $1 \leq i < j \leq m$ , is called a Diophantine  $m$ -tuple with the property  $D(n)$  or simply a  $D(n)$ -set.  $D(1)$ -sets are known as Diophantine  $m$ -tuples. The first Diophantine quadruple, the set  $\{1, 3, 8, 120\}$  was found by Fermat. He, Togbe and Ziegler proved recently that there does not exist a Diophantine quintuple. On the other hand, it is known that there exist infinitely many rational Diophantine sextuples. When considering  $D(n)$ -sets, usually an integer  $n$  is fixed in advance. However, we may ask if a set can have the property  $D(n)$  for several different  $n$ 's. For example,  $8, 21, 55$  is a  $D(1)$ -triple and  $D(4321)$ -triple. In a joint work with Adzaga, Kreso and Tadic, we presented several families of Diophantine triples which are  $D(n)$ -sets for two distinct  $n$ 's with  $n \neq 1$ . In a joint work with Petricevic we proved that there are infinitely many (essentially different) quadruples which are simultaneously  $D(n_1)$ -quadruples and  $D(n_2)$ -quadruples with  $n_1 \neq n_2$ . Moreover, the elements in some of these quadruples are squares, so they are also  $D(0)$ -quadruples. E.g.  $\{54^2, 100^2, 168^2, 364^2\}$  is a  $D(8190^2)$ ,  $D(40320^2)$  and  $D(0)$ -quadruple. In this talk, we will describe methods used in constructions of mentioned triples and quadruples. We will also mention a recent joint work in with Kazalicki and Petricevic on  $D(n)$ -quintuples with square elements (so they are also  $D(0)$ -quintuples). There are infinitely many such quintuples. One example is a  $D(480480^2)$ -quintuple  $225^2, 286^2, 819^2, 1408^2, 2548^2$ .



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Alle Interessierten sind herzlich eingeladen.