Automorphisms of extremal lattices and codes

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Using invariant theory of finite complex matrix groups, Andrew Gleason has shown in his ICM talk in Nice 1970, that the minimum distance of a doubly-even self-dual binary code of length n cannot exceed $4 + 4\lfloor \frac{n}{24} \rfloor$. A similar bound has been proven by Siegel for even unimodular lattices of dimension n, where the minimum is always $\leq 2 + 2\lfloor \frac{n}{24} \rfloor$. Lattices and codes achieving equality are called **extremal**. Of particular interest are extremal lattices and codes in the "jump dimensions" - the multiples of 24, where one only knows 2 extremal codes (of length 24 and 48) and 6 extremal lattices (of dimension 24, 48 (4 lattices) and 72).
A very intensively studied question is the existence of an extremal code of length 72. The main strategy to construct such a code is to classify all extremal codes with a given automorphism. This leads to very nice explicit applications of basic theorems in group theory and representation theory. Combining the theoretical and computational results of several authors we know that if such an extremal code of length 72 exists, then its automorphism group has order ≤ 5.
I will also address the similar question for lattices.