Pairs of polyhedra connected by a piecewise-linear bijection appear in different fields of mathematics. The model examples of this situation are the order and chain polytopes introduced by Stanley in 1986, whose defining inequalities are given by a finite partially ordered set. A piecewise-linear transfer map lets the two polytopes share properties like number of vertices and their Ehrhart polynomial despite not being combinatorially equivalent. Further motivating examples provided by representation theory and enumerative combinatorics will be discussed in the talk. Moving towards a common framework capturing this phenomenon, we report on previous work on marked poset polytopes by Ardila, Bliem and Salazar as well as Jochemko and Sanyal. We present joint work with Xin Fang and Ghislain Fourier on a continuous family of polytopes interpolating between marked order and chain polytopes. Beyond partially ordered sets, we present joint work in progress with Raman Sanyal on transfer maps for distributive polyhedra.